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$$u = 2^{\frac{1}{2}}q^{\frac{1}{8}}(1-q+2q^2-3q^3+4q^4),$$
$$v_0 = 2^{\frac{1}{2}}q^{\frac{4}{56}}(1-q^{\frac{1}{7}}+2q^{\frac{2}{7}}-3q^{\frac{3}{7}}+4q^{\frac{4}{7}}).$$

We find also for the first combination

$$U = i2^27^{\frac{1}{2}}q^{\frac{1}{14}}\left(1-\frac{1-i\sqrt{7}}{2}q^{\frac{2}{7}}+q^{\frac{4}{7}}\right),$$

whence

$$a_7 = i2^{12}7^{\frac{7}{2}},\ a_5 = 0,\ a_3 = -i2^47^{\frac{5}{2}}\frac{1-i\sqrt{7}}{2},\ a_6 = 2^87^4\left[1-\left(\frac{1-i\sqrt{7}}{1}\right)\right].$$

We deduce the second combination from the first in replacing i by $-i$.

The following is extracted from Todhunter's Theory of Equations, sections 340, 341.

"The general equation of the fifth degree can always be reduced to one of the following forms:

"$x^5+px+q=0$; $x^5+px^2+q=0$; $x^5+px^3+q=0$; $x^5+px^4+q=0$. [Due to Mr. Jerrard or to E. S. Bring. Quarterly Journal of Mathematics, Vol. VI.]

"Mr. Jerrard considered that the algebraical solution of equations of the 5th degree could be effected; his proposed method formed the subject of an enquiry by Sir W. R. Hamilton in the *Reports of the British Association*, Vol. VI. Most mathematicians admit that ABEL has demonstrated the impossibility of the algebraical solution of equations of a higher degree than the fourth. An abstract of Sir W. R. Hamilton's exposition of *Abel's* argument will be found in the *Quarterly Journal of Mathematics*, Vol. V. A simpler demonstration due to *Wantzel* will be found in *Serret's Cours d' Algebre Supérieure*." [See ANALYST, p. 65–70, Vol. IV.] "An *Essay* on the resolution of algebraical equations by the late *Judge Hargreave* has been recently published: the results arrived at are to some extent at variance with those of *Abel* and *Sir W. R. Hamilton*."

TO DRAW A CIRCLE TANGENT TO THREE GIVEN CIRCLES.

BY ISAAC H. TURRELL, CINCINNATI, OHIO.

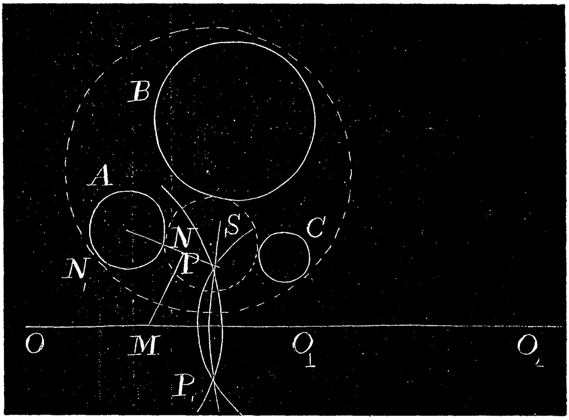
THIS problem is a celebrated one in the history of pure geometry. When or by whom it was first proposed I am unable to ascertain; but the best mathematicians of modern times have not thought it unworthy their special attention. In Salmon's Conics (5th edition) not less than four different solutions are given. In the 8th volume of Smithsonian Contributions to Knowledge, Major Alvord, by a different and very elegant method, solves, not only this, but the more general problem— "To draw a sphere tangent to four given spheres."

The following solution so far as the writer is aware is original. It depends upon the following elementary propositions.

I. A circle orthogonal to two tangent circles passes through their point of contact.

II. All the circles touching two given circles externally, are cut orthogonally by an auxiliary circle whose center is the external center of similitude of the two circles.

III. An axis of similitude of three given circles, and the radical axis of a pair of circles touching them are coincident.



Let us first take the case in which the circles O, O_1, O_2, which are drawn auxiliary (Pr. II) to the pairs AB, AC and BC, respectively, meet in two points P, P_1. Since these auxiliaries have their centers on the radical axis (Prop. III) of the req'd pair S, S_1, which they cut orthogonally, they must pass through their limiting points; hence P, P_1 are the limiting points, or infinitessimal circles, of the system determined by S, S_1.

To find the points of tangency on A, draw the radical axis of P and A meeting OO_1 in M. The circle M, passing through P, evidently cuts the three circles S, S_1 and A orthogonally; hence (Prop. I) it passes through their points of contact N, N_1. In a similar manner the points of tangency of S, S_1 with B and C can be obtained.

It may happen that the auxiliaries O, O_1, O_2, will not intersect; but since they belong to a system complemental to that of S, S_1, the latter will pass through their limiting points, which will lie on the line OO_1.

In this case, which a slight change in the diagram will illustrate, find the limiting points, P, P_1 of the system O, O_1, O_2, through which draw any circle V. This circle will belong to the system S, S_1. From M, the intersection of the radical axis of V and A with the line OO_1, draw a circle cutting V orthogonally; it will evidently cut S, S_1, A orthogonally, and pass through their points of contact N, N_1, as in the former case.

Repeating this process with the other three axes of similitude we can determine the other six touching circles.

This solution can be readily extended to include Gen'l Alvord's problem on the tangencies of Spheres.

SUMMATION OF TWO SERIES.

BY PROF. D. TROWBRIDGE, WATERBURGH, NEW YORK.

REQUIRED the sums of the following series:

$$S_x^{(n)} = (a+r)^n+(a+2r)^n+(a+3r)^n+\ \ldots\ +(a+rx)^n, \qquad (1)$$

$$S_x^{(n)} = (a+r)^n+(a+3r)^n+(a+5r)^n+\ \ldots\ +(a+2rx-r)^n. \qquad (2)$$

In these series x is the number of terms, and hence, when $x = 0$, the sums will equal nothing. It is plain that the sum of either series will be a function of x; that is it will depend on x in such a way that when $x=0$, the sum will be 0; and we easily see that the sum will involve the powers of x from x^{n+1} down to x; we say the $(n+1)$th power, because the expansion of the last term will give the nth power, and since there are x terms, all similar, there will be the $(n+1)$th power of x, at least. We may therefore assume

$$(a+r)^n+(a+2r)^n+\ldots\ldots+(a+rx)^n=A_1x^{n+1}+A_2x^n+\ldots+A_{n+1}x, \qquad (3)$$

$$(a+r)^n+(a+3r)^n+\ldots+(a+2rx-r)^n=B_1x^{n+1}+B_2x^n+\ldots+B_{n+1}x. \qquad (4)$$

In these series let $x+1$ be written for x—for they are true for at least all positive integral powers of x—and we shall have

$$\begin{aligned}(a+r)^n+(a+2r)^n+\ldots\ldots+(a+rx)^n+(a+rx+r)^n &= A_1(x+1)^{n+1}\\ &+A_2(x+1)^n+\ldots\ldots+A_{n+1}(x+1), \qquad (5)\end{aligned}$$

$$\begin{aligned}(a+r)^n+(a+3r)^n+\ldots+(a+2rx-r)^n+(a+2rx+r)^n &= B_1(x+1)^{n+1}\\ &+B_2(x+1)^n+\ldots\ldots+B_{n+1}(x+1). \qquad (6)\end{aligned}$$

Now substitute (3) from (5) and (4) from (6) and we have

$$(a+rx+r)^n=A_1[(x+1)^{n+1}-x^{n+1}]+A_2[(x+1)^n-x^n]+\ldots+A_{n+1}, \qquad (7)$$

$$(a+2rx+r)^n=B_1[(x+1)^{n+1}-x^{n+1}]+B_2[(x+1)^n-x^n]+\ldots+B_{n+1}. \qquad (8)$$

Now make $a+r = b$, and $2r=r'$ in (8) and the two series will have the same form. We can, therefore, easily find the B's from the A's.